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Finite element model updating taking into account the uncertainty on the modal parameters estimates

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Abstract

Model updating is a common method to improve the correlation between finite element (F.E.) models and measured data. F.E. model updating is a technique that is used to identify and correct uncertain modelling parameters that leads to better predictions of the dynamic behavior of a target structure. A number of approaches to the problem exist, based on the type of parameters that are updated and the measured data that is used. This paper concentrates on an updating method based on measured modal data. The conventional F.E. updating techniques are extended and adapted in order to take into account the uncertainty on the estimated modal parameters to be updated. The effectiveness of the suggested procedure is tested on a case study.

1. Introduction

Finite element (F.E.) model updating is an inverse problem to identify and correct uncertain modelling parameters that leads to better predictions of the dynamic behavior of a target structure. The procedure of updating a F.E. model uses measurement data as exact reference data in order to update the chosen modelling parameters. The experimentally measured resonance frequencies and mode shapes are considered to be exact and the modelling parameters are updated in order to minimize the differences between experimental and numerically calculated resonance frequencies and mode shapes.

In reality, this assumption of 100% accurate experimental data results is not completely true. When performing measurements and estimating poles in a repetitive manner, there is always uncertainty in the measured and estimated resonance frequencies that can be characterized by its mean value and standard deviation. Once these statistical properties are known, one can implement these properties in the F.E. updating process in order to take into account the measurement uncertainties.

Examples on this issue can be found in scientific literature: Farrar et al. (Ref. [1]) discuss the prediction of the resonance frequencies for the modal response of laminated composite plates. Modal tests are conducted on a population of eight nominally identical plates. The test-to-test variability and unit-to-unit variability are fully characterized using sample statistics and principal component analysis. Test measurements are compared

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to predictions, and statistics of experimental variability and test–analysis correlation are also listed. Various independent sources of modelling uncertainty as well as the test–analysis correlation error model are needed to assess the prediction accuracy of the composite model. Zang et al. (Ref. [2]) apply the robust design approach to the dynamics of a tuned vibration absorber due to parameter uncertainty. The objective is to minimize the displacements of the main system over a large range of excitation frequencies, despite uncertainty in the mass and stiffness properties of the main system. Shin and Cho (Ref. [3]) describe the production problem of bare silicon wafers where production runs exhibit problems maintaining the dimensions, surface quality and flatness of the housing. The authors use fitted response functions for the mean and standard deviation of the coating thickness.

The focus of this paper will be partly directed toward statistically derived response surface models. Response surface methods may be employed with low effort and have the potential to be applied to both linear and nonlinear problems. Because the total design space is often prohibitively large, methods have been developed in the literature to efficiently explore it. In the case of response surface metamodels, design of experiments methods (often called response surface methods) are employed. Design of experiments is a technique to determine the location of the sampling points. There are several versions for design of experiments available in literature (Refs. [4–6]). These techniques have in common that they are trying to locate the sampling points such that the space of random input variables is explored in a most efficient way, meaning obtaining the required information with a minimum number of sampling points. Metamodels may then be fitted to these intelligently chosen data points using standard multiple regression methods, resulting in a polynomial model that relates input parameters to output features. While these models are empirical in nature, they rely on the expertise of the experimenter for assignment of model input parameters and choice of appropriate output feature(s).

Fuzzy arithmetic is another technique that can be applied to model input uncertainty of F.E. models. The shape of the membership function can be derived from expert knowledge or practical measurements. These membership functions are probabilistic distribution functions that denote if an input is possible, impossible or something in between. Once the input uncertainty has been modelled, the output uncertainty can be computed by using fuzzy arithmetic (Ref. [7]). Application to structural dynamics problems is not straightforward, as fuzzy solvers are either slow or non-existing and inversion of a fuzzy matrix is not (yet) possible. Numerical approximations have therefore been developed, such as interval arithmetic (Ref. [8]). Each fuzzy number is represented by a set of intervals at different levels of membership and interval arithmetic can then be applied to the input interval extremes to find an interval representation of the output. Unfortunately, this often leads to overestimation of the output uncertainty (Refs. [9,10]).

A hybrid method, consisting of F.E. modelling, response surface methods, the first-order reliability method and an iterative linear interpolation scheme is proposed by Huh and Haldar (Ref. [11]) to estimate the reliability of real nonlinear structures. The authors propose a nonlinear, F.E.-based algorithm incorporating the uncertainties in the random variables in order to estimate the reliability of realistic structural systems. The authors consider a hybrid method, consisting of stochastic F.E. modelling, response surface methodology and the first-order reliability method. They choose iterative combinations of second-order polynomials (with and without interaction terms) and saturated and central composite designs in order to achieve efficiency and accuracy. In this reference, the F.E. method was combined with probabilistic concepts from the Monte Carlo simulation to compute failure probability in structural reliability issues. Therefore, the question of how safe the structure is in reality can be answered by considering the stochastic nature of uncertain structural parameters such as element properties, loads, and geometry. These uncertain parameters are all represented by stochastic or random variables that is not of a fixed value, but by a probability distribution function that characterizes the behavior and safety of a structure.

This paper describes the development of an automated model updating procedure using one updating parameter. The proposed procedure will then be extended to multiple updating parameters, taking into account the measurement uncertainty. Also the influence of the mesh will be briefly discussed. Frequency response function (FRF) measurements were performed on an aluminum test plate with known geometrical and material properties. From these FRF measurements, the resonance frequencies (poles) were estimated. A F.E. model for the test plate is created and used as function of one or more modelling parameters. The experimental results on the simple test case are discussed and the automated model updating technique is described in detail and updated parameters from the F.E. model are compared to the original ones. The results are explained and discussed. There is only limited scientific literature to be found that addresses the hybrid topic of F.E. updating taking into account both measurement uncertainty and response surface modelling.

The advantages of the proposed updating technique presented in this research paper are:

- The F.E. model is replaced by a metamodel, or response surface model in order to decrease calculation time.
- Development of an automated model updating procedure using multiple updating parameters.
- The measurement uncertainty is taken into account by means of statistical measurement properties.
- These statistical properties are directly implemented in the F.E. optimization algorithm.

The main advantage of the proposed updating technique lies in the fact that the link with the F.E. model is kept. Other response surface techniques start from a design of experiments to choose the surface response points and afterwards only work with the response surface model and neglect the F.E. model. In the suggested updating approach, the new response points are defined straight from the F.E. model output and are not defined through a design of experiments.

2. Comparison between test and F.E. resonance frequencies

2.1. Experimental results

Because experimental data is needed to validate a F.E. model, measurements were performed on a simple aluminum test plate with dimensions $0.39\,\mathrm{m} \times 0.11\,\mathrm{m} \times 0.0119\,\mathrm{m}$. The aluminum plate was excited with an impact hammer at an outer point of the plate and measured at a diagonal point on the other side of plate. In total, the setup contained three measurement channels: one channel for the hammer force cell, one driving point acceleration measurement and a response acceleration measurement point on the other side of the plate, diagonal with respect to the hammer excitation point. FRF measurements are performed in free–free boundary conditions, using the hammer excitation technique and the modal parameters (modal frequencies, damping and mode shapes) are estimated using LMS Cada-X software (Ref. [12]). Fig. 1 shows a typical FRF measured on the aluminum test plate. For a frequency range between 0 and 6000 Hz, 10 modes are selected. The modal parameters are estimated using the time domain MDOF estimator (Ref. [13]).

2.2. Modal data comparison

A F.E. model of the aluminum test plate with approximately correct dimensions and aluminum material properties is created using Femlab software (Ref. [14]), that will be used together with the test data in the updating procedure. The advantage of using Femlab software is the fact that an immediate input–output interaction with Matlab is possible and the created F.E. model can be stored as a Matlab file and altered as a function of certain updating parameters afterwards. In Femlab, a default meshing is applied and the resonance frequencies are calculated, using a direct eigenvalue solver. As a first result, one can make a comparison between the measured and calculated resonance frequencies of the test model and F.E. model (Table 1).

One notices that the relative frequency differences between measured (test) modes and F.E. modes are already quite small and no more than 6%. Of course, the fact that the aluminum test plate has well-known dimensions and material properties will make the first F.E. results already quite accurate. One can still optimize one or more updating parameters in order to make the frequency differences as small as possible. As a next step, one can investigate the influence of the mesh density and element size on the calculated F.E. modal frequencies and accordingly choose a satisfying element mesh.

2.3. Influence of the mesh

Before creating and evaluating an updating procedure to make the difference between test and F.E. modal frequencies as small as possible, one can examine the influence of the applied mesh density on the modal

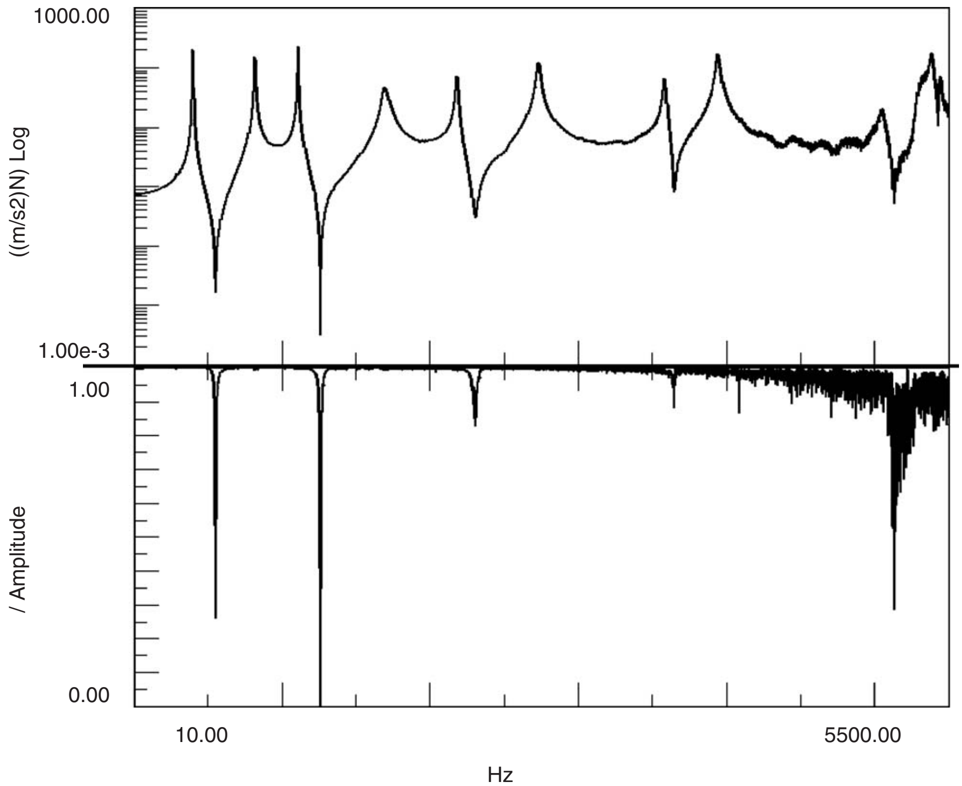

Fig. 1. A typically measured FRF (upper) together with its measurement coherence (lower).

Table 1
Comparison between estimated test and calculated F.E. modes

Mode	Test (Hz)	F.E. (Hz)	Relative difference (%)
1	404.33	410.61	1.55
2	820.51	857.42	4.50
3	1115.16	1139.38	2.17
4	1702.86	1796.10	5.48
5	2185.34	2244.94	2.73
6	2735.45	2885.29	5.48
7	3589.07	3728.00	3.87
8	3948.22	4195.23	6.26
9	5065.04	5208.00	2.82
10	5356.18	5542.11	3.47

frequencies. Other approximations will be neglected. As a result, a meshing density will be chosen according to the difference in frequencies between test and F.E. modes (Ref. [15]). For the test case of the simple plate, different mesh densities are used for the calculation of the F.E. modes. The calculations are presented in Table 2.

Table 2 shows an examination of the mesh convergence for the first 10 plate resonance frequencies. Different F.E. calculation runs have been performed to check whether the mesh is converged when using 12 110 mesh elements, as indicated and used for the results presented in the paper. It is clear that the F.E. model shows convergence when using 12 110 mesh elements. Due to the fact that the calculation time for each F.E. eigenfrequency calculation and thus for each iteration step in the optimization routine increases when using more elements, it is decided by the authors to use an F.E. mesh containing 12 110 elements.

Table 2
Comparison between estimated test and calculated F.E. modes using 5810, 12 110, 16 982 and 24 017 elements

Test modes (Hz)	F.E. 5810 (Hz)	F.E. 12 110 (Hz)	F.E. 16 982 (Hz)	F.E. 24 017 (Hz)
404.33	409.70	409.59	409.60	409.56
820.51	846.21	843.84	843.80	843.72
1115.16	1130.65	1128.42	1128.15	1128.04
1702.86	1762.40	1757.37	1757.21	1757.03
2185.34	2209.58	2206.21	2206.05	2205.83
2735.45	2813.04	2803.82	2803.49	2803.21
3589.07	3619.31	3606.02	3605.70	3605.34
3948.22	4053.70	4032.3	4031.54	4031.14
5065.04	5085.43	5062.9	5061.87	5061.36
5356.18	5434.55	5410.85	5409.34	5408.80

3. Updating procedure using one updating parameter

In this section, a method will be developed and explained in order to update the F.E. modal parameters based on one single updating parameter, taking into account measured modal frequencies as reference data. In the F.E. software package Femlab, the F.E. model file can be exported as a Matlab file where one can modify the model file as a function of one or more parameters. To start, one can consider the plate length (L) as the first updating parameter. A nonlinear least squares problem is solved in order to minimize a cost function consisting of the sum of the squares of the frequency differences between estimated (test) and calculated (F.E.) resonance frequencies. In particular, the discipline of F.E. model updating is a nonlinear parameter estimation problem, since the modal data are nonlinear functions of the unknown structural properties such as the Young's modulus, the spring stiffness, etc. (Ref. [16]).

$$\min_{\boldsymbol{\theta}} \ell(\boldsymbol{\theta}) = \min_{\boldsymbol{\theta}} \sum_i r_i^2(\boldsymbol{\theta}) \tag{1}$$

with

$$r_i(\boldsymbol{\theta}) = f_i^{\text{measured}}(\boldsymbol{\theta}) - f_i^{\text{F.E.}}(\boldsymbol{\theta}), \tag{2}$$

where $\boldsymbol{\theta}$ is a vector, $r(\boldsymbol{\theta})$ a function that returns a vector value, $\ell(\boldsymbol{\theta})$ the cost function to be minimized with $f_i^{\text{measured}}(\boldsymbol{\theta})$ the ith measured target eigenfrequency and $f_i^{\text{F.E.}}(\boldsymbol{\theta})$ the ith calculated F.E. eigenfrequency.

For solving the nonlinear least squares problem, one needs to specify an upper and lower boundary value for the plate length (Ref. [17]). The least squares problem is solved with the used boundary values as starting values. A regression algorithm with increasing regression order is used to calculate the regression coefficients of the interpolation polynomial in order to fit an interpolation curve through the already known (numerically calculated) modal frequencies. For the fitting of one updating parameter, one needs two starting values in order to start from a linear regression fit for the updating parameter. In a first step, the cost function will be evaluated based on the initially calculated parameter value eigenfrequencies and the measured eigenfrequencies. By calculating a higher order fit through the already known values of updating parameter L and the calculated F.E. frequencies, respectively, the F.E. model is replaced by an interpolation polynomial and needs to be calculated only a few times. The updating algorithm will stop when the difference in F.E. eigenfrequency after a parameter update becomes smaller than a user-defined threshold.

The cost function to be minimized has the following form (Refs. [16,17]):

$$\min_L \ell(L) = \sum_i r_i^2(L), \tag{3}$$

where

$$r_i(L) = f_i^{\text{measured}}(L) - f_i^{\text{interp}}(L) \tag{4}$$

with $f_i^{\text{measured}}(L)$ the ith measured target eigenfrequency and $f_i^{\text{interp}}(L)$ the ith interpolated F.E. eigenfrequency based on the interpolation polynomial coefficients (a_{ij}), the already known F.E. eigenfrequencies and respective length values:

$$f_i^{\text{interp}}(L) = \sum_{j=0}^{n} a_{ij} L^j. \quad (5)$$

To summarize: the updating algorithm consists of three steps:

(1) Calculate interpolation polynomial between calculated resonance frequencies and their respective updating parameter values (L).
(2) Minimize cost function based on measured and interpolated resonance frequencies.
(3) Solve nonlinear least squares problem to estimate new parameter value L, based on the defined cost function, measured (target) and interpolated eigenfrequencies and a starting parameter value.

The updating algorithm will now be used to optimize the updating parameter L (plate length) and to find the accurate length of the test plate (Table 3). The measured length of the test plate is 0.39 m. Starting from two boundary values ($L = 0.3, 0.46$), the F.E. eigenfrequencies of the test plate are calculated. Fig. 2 shows the linear interpolation curve for the first resonance frequency (λ_1) as a function of the plate length. Between these two boundary values, a linear interpolation value for the plate length is used to solve the nonlinear least squares problem of minimizing the cost function. For each resonance frequency, a different interpolation polynomial is calculated, taking into account the interpolation order, the values of L and respective eigenfrequencies. The cost function reaches a minimum for a value of $L = 0.3983$ m in the first iteration step. In the next iteration step, the resonance frequencies for the next resulting plate length are calculated, the order of the interpolation polynomial is increased by 1 and a second-order interpolation of the F.E. model is used to solve the nonlinear least squares problem of minimizing the cost function (Fig. 3).

Table 3
Updating plate length (L) using different meshes (272, 920 and 12 110 mesh elements)

Mesh density	L updated (m)	Relative difference (%)
Finest mesh (12 110 el.)	0.3931	0.79
Fine mesh (920 el.)	0.3947	1.21
Default mesh (272 el.)	0.3983	2.13

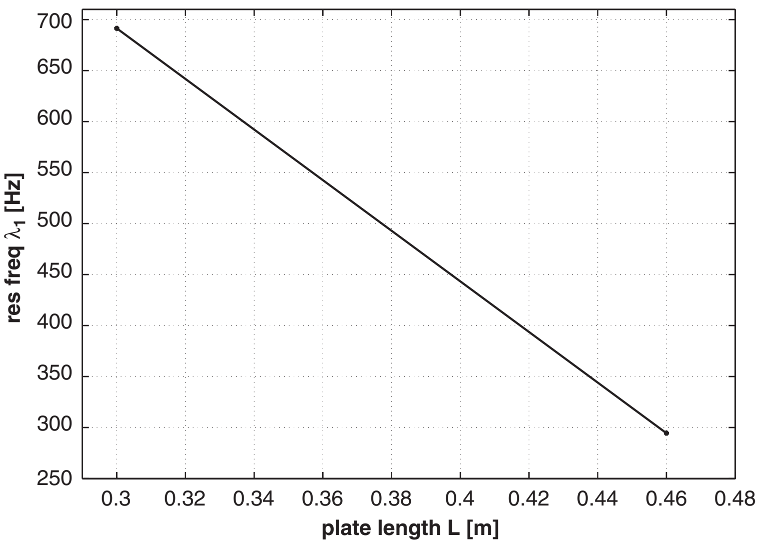

Fig. 2. Linear interpolation of first plate resonance frequency between boundary values.

From previous results it is clear that the updating procedure works for updating the parameter L. The updating procedure generates results within a 2% tolerance. One can also conclude that the updating algorithm gives the best result (smallest relative difference) when using the finest mesh density. The less F.E.s one uses, the more the plate length that is overestimated. When using a reduced number of elements, there tends to be an overestimation of the F.E. model stiffness. A stiffness increase leads to an increase of modal frequencies. When optimizing the plate length, the updating algorithm will compensate for this stiffness overestimation by making the test plate longer than the actual plate length. One can also optimize other model parameters, for instance the test plate width (W) and Young's modulus (E), using the updating procedure of optimizing one parameter.

From previous results (Tables 4 and 5) it is again clear that the updating procedure works for updating the parameters Young's modulus (E) and plate width (W). The updating procedure generates results within a 5% tolerance. In this case, one can also conclude that the updating algorithm gives the best result (smallest error) when using the finest mesh density. In this case, the relative difference decrease is higher when updating the Young's modulus because this is the parameter with the highest uncertainty. The plate length and width are already well-known. Thus updating these geometric parameters will result in a smaller benefit than updating the most uncertain parameter E which is less known.

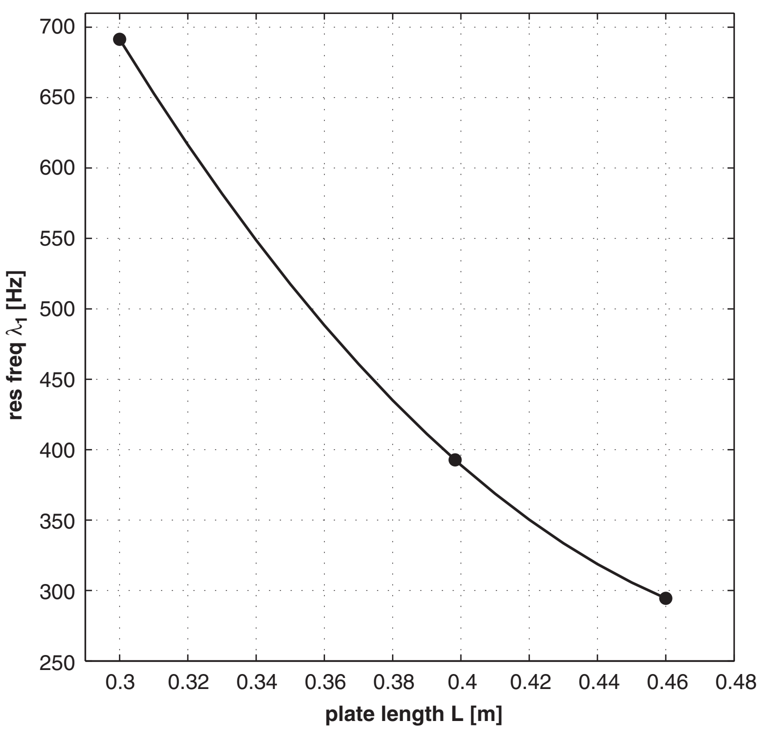

Fig. 3. Second-order interpolation of first plate resonance frequency.

Table 4
Updating plate Young's modulus (E) using different meshes (272, 920 and 12 110 mesh elements)

Mesh density	E updated (Pa)	Relative difference (%)
Finest mesh (12 110 el.)	6.92E + 10	1.13
Fine mesh (920 el.)	6.82E + 10	2.54
Default mesh (272 el.)	6.64E + 10	5.17

Table 5
Updating plate width (W) using different meshes (272, 920 and 12 110 mesh elements)

Mesh density	W updated (m)	Relative difference (%)
Finest mesh (12 110 el.)	0.1136	3.27
Fine mesh (920 el.)	0.1142	3.82
Default mesh (272 el.)	0.1147	4.27

4. Taking into account the measurement standard deviation

The least squares problem formulation allows the residuals to be weighted separately corresponding to their importance and amount of noise. The ability to weight the different data sets gives the method its power and versatility, but at the same time requires engineering insight to provide the correct weights. In a weighted least squares problem, the following minimization problem is solved (Ref. [16]):

$$\min_{\boldsymbol{\theta}} r^{\mathrm{T}}(\boldsymbol{\theta})\mathbf{W}r(\boldsymbol{\theta}) = \min_{\boldsymbol{\theta}} \|\mathbf{W}^{1/2}r(\boldsymbol{\theta})\|_2^2 \tag{6}$$

with $\mathbf{W}$ the weighting matrix. If the weighting matrix is a diagonal matrix, i.e. $\mathbf{W} = \mathrm{diag}(\ldots, w_j^2, \ldots)$, Eq. (6) can be equivalently written as

$$\min_{\boldsymbol{\theta}} r^{\mathrm{T}}(\boldsymbol{\theta})\mathbf{W}r(\boldsymbol{\theta}) = \min_{\boldsymbol{\theta}} \sum_{j=1}^{m} [w_j r_j(\boldsymbol{\theta})]^2, \tag{7}$$

where w_j is the weighting factor of the residual r_j.

The weighting factors can also be defined based on the statistical properties of the measurements. Friswell and Mottershead (Ref. [15]) propose to use a diagonal weighting matrix whose elements are given by the reciprocals of the variance of the corresponding measurements:

$$\mathbf{W} = [\mathrm{diag}(\sigma_1^2, \ldots, \sigma_j^2)]^{-1} \quad \text{or} \quad w_j^2 = \frac{1}{\sigma_j^2} \tag{8}$$

with σ_j^2 the variance of the measured quantity as used in residual r_j.

If various sets of measurement data at different time steps are available, the weighting of the modal data can be based on the statistical properties of the measurements (mean value and standard deviation of the resonance frequencies) (Table 6). If the m measurements are not equally reliable (different standard deviation), it is convenient to give them different weights, whereby relatively higher weights are assigned to the measured resonance frequencies that are trusted more. To investigate the accuracy and repetition of the estimated experimental modal frequencies, FRF measurements on the same aluminum test plate are repeated 10 times and for each measurement the modal frequencies are estimated, using LMS Cada-X software. From these modal frequency estimations, the statistical properties (mean value and standard deviation) are calculated and used as an inverse weighting factor in the updating algorithm. The more noisy the data, the higher the standard deviation and the less weight is given to the modal frequency in the updating process.

From the repeated measurements, the mean value and standard deviation of the first 10 resonance frequencies are calculated. Fig. 4 shows the standard deviation is not increasing monotonic with higher frequencies, as one might expect. Modes 9 and 10 show a remarkable increase in standard deviation. This can be explained by the fact that the hammer excitation force puts less energy in the higher frequency region due to the fact that the hammer tip impact determines the frequency range of the excitation force. When the excitation force is lower for a certain frequency range, the FRFs are more noisy in this frequency range and the uncertainty of the pole estimation becomes higher. As a next step, the updating algorithm needs to be modified in order to include the measurement uncertainty. This is realized by modifying the cost function to be minimized. The standard deviation must be added to the cost function as extra weighting. From Eqs. (7) and (8) it is shown that the inverse of the standard deviation can be used as a weighting factor in the cost function,

Table 6
Statistical properties of the test modes

Mode no.	Mean (Hz)	Standard deviation (Hz)
1	403.81	0.51
2	820.55	0.10
3	1115.14	0.10
4	1703.72	1.01
5	2185.41	0.36
6	2734.84	0.67
7	3589.51	0.71
8	3948.29	0.27
9	5062.26	3.70
10	5370.27	8.38

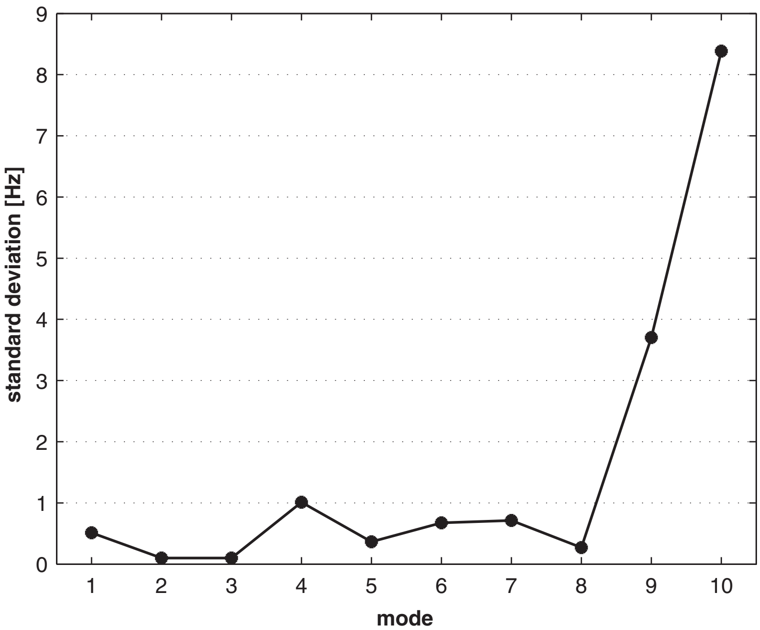

Fig. 4. Standard deviation of first 10 plate modes.

whereby relatively higher weights are assigned to the measured resonance frequencies that are trusted more (Fig. 5).

Taking this into account, the original cost function (4), used in the updating algorithm is now modified into:

$$\min_{\boldsymbol{\theta}} \ell(\boldsymbol{\theta}) = \sum_i f_i^2(\boldsymbol{\theta}), \tag{9}$$

where

$$f_i(\boldsymbol{\theta}) = \frac{f_i^{\text{measured}}(\boldsymbol{\theta}) - f_i^{\text{interp}}(\boldsymbol{\theta})}{\sigma_i^{\text{measured}}} \tag{10}$$

with f_i^{measured}, the calculated mean value of the measured eigenfrequencies and $\sigma_i^{\text{measured}}$ the standard deviation calculated from 10 repetitive measurements for the ith mode.

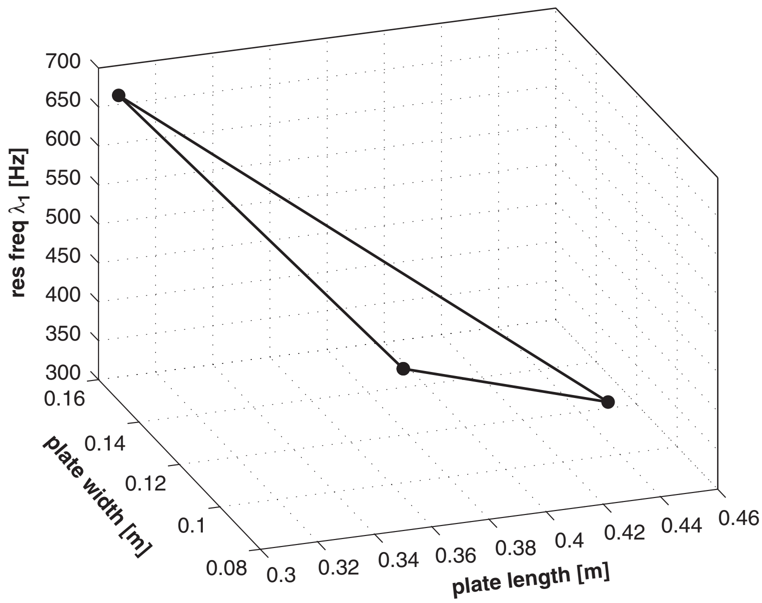

Fig. 5. Linear interpolation of first plate resonance frequency between boundary values.

Table 7
Updating plate length (L upd) and relative difference with and without taking into account the measurement uncertainty

	L upd (m)	Rel. diff. (%)	L upd std. (m)	Rel. diff. (%)
Finest mesh (12 110 el.)	0.3931	0.79	0.3908	0.21
Fine mesh (920 el.)	0.3947	1.21	0.3937	0.95
Default mesh (272 el.)	0.3983	2.13	0.3977	1.97

Table 8
Updating plate width (W) and Young's modulus (E) with and without taking into account the measurement uncertainty

	No std.	Relative difference (%)	w/std.	Relative difference (%)
W	0.1135 m	3.18	0.1126 m	2.36
E	6.92E + 10 Pa	1.14	6.92E + 10 Pa	1.14

The modified updating algorithm will now be used to optimize the updating parameter L (plate length) and to find the accurate length of the test plate, taking into account the uncertainty on the modal frequencies. The measured length of the test plate is 0.39 m.

From Table 7 it is clear that taking into account the measurement uncertainty in the form of the measurement standard deviation gives a better prediction for the plate length. The more reliable (better estimated) resonance frequencies have a higher weighting in the updating algorithm which leads to more accurate results. One may consider that the effect of the parameter length on the global plate stiffness is similar to the parameter Young's modulus. Because both parameters influence the global stiffness, this might lead to a discrepancy in resonance frequency shift and might cause less accurate updating results. On the other hand, the plate width has a smaller effect on the global plate stiffness. This is why the simultaneous updating of plate width and Young's modulus is examined. The same results and conclusions are found in Table 8 when updating the parameters plate width (W) and Young's modulus (E).

5. Updating procedure using multiple updating parameters

5.1. Description

Once the updating procedure using one updating parameter is working and giving feasible results, one can now try to adapt and extend the procedure for updating more than one parameter, in order to see the effects of changing multiple parameters simultaneously on the F.E. model updating. Extending the updating algorithm for two or more updating parameters requires a slightly different approach. An array of all order coefficients up to a certain order (e.g. order 4) is defined and used as regression coefficients for the development of the regression polynomial. For solving the nonlinear least squares problem, one needs to specify two boundary values for each updating parameter where, in between, the correct value of the updating parameter is situated. When updating two or more parameters, combinations of the boundary values for all updating parameters must be specified. The Matlab F.E. model stack containing the F.E. model data as a function of the updating parameters is solved for the initial updating parameter boundary value combinations. The output consists of the eigenfrequency values with respect to the initial updating parameter values.

For the updating of three parameters, one needs four starting value combinations. In general, for the updating of (n) parameters, one needs ($n+1$) starting value combinations. An internal function with input order coefficients, updating parameters and eigenfrequency values is called for the calculation of the regression coefficients in a least squares sense:

$$f^{\text{interp}}(\boldsymbol{\theta}) = \mathbf{X}(\boldsymbol{\theta}).\mathbf{c} \tag{11}$$

with $\mathbf{X}$ a matrix containing combinations of parameter values with order coefficient powers, $f^{\text{interp}}(\boldsymbol{\theta})$ the F.E. interpolated frequencies and $\mathbf{c}$ the calculated multiple regression coefficients in a least squares sense. Afterwards, a cost function needs to be minimized. One can use again the same cost function as for the one updating parameter case, taking into account the measurement uncertainty (10). The updating algorithm will stop if a smaller difference in updating parameters is achieved, between calculated and measured modal frequencies, than specified by the user. In the case of updating two parameters (e.g. plate length L and width W) the procedure yields the following intermediate results. For two parameters, three starting boundary value combinations are specified by the user and listed in Table 9.

For these boundary value combinations, the F.E. eigenfrequencies and the resulting cost function are calculated. The cost function consists of the sum of the squares of the differences between measured and calculated eigenfrequencies. An array of all order coefficients up to a certain order (e.g. order 4) is defined and used as regression coefficients for the development of the regression polynomial (Table 10).

Table 9
Boundary value combinations for parameters L and W specified by the user

Boundary value combination	L (m)	W (m)
1	0.35	0.08
2	0.45	0.12
3	0.3	0.15

Table 10
Regression coefficients n_1 and n_2 used for the development of the regression polynomial

Regression order	0	1		2			3				4				
n_1	0	1	0	2	1	0	3	2	1	0	4	3	2	1	0
n_2	0	0	1	0	1	2	0	1	2	3	0	1	2	3	4

A matrix **X** is built, consisting of combinations of regression order coefficients and updating parameters. In the case of a linear regression, **X** gives:

$$\mathbf{X} = \begin{pmatrix} 1 & 0.35 & 0.08 \\ 1 & 0.45 & 0.12 \\ 1 & 0.30 & 0.15 \end{pmatrix}. \tag{12}$$

Because the algorithm is starting from a linear regression, one can recognize the boundary values as matrix elements. The matrix elements are the boundary values to the power of the first six elements of the order coefficient array (linear regression order 1). The first row of matrix (12) consists of the elements:

$$\mathbf{X}(1,1) = 0.35^{\circ} \times 0.08^{\circ} = 1,$$
$$\mathbf{X}(1,2) = 0.35^{1} \times 0.08^{\circ} = 0.35,$$
$$\mathbf{X}(1,3) = 0.35^{\circ} \times 0.08^{1} = 0.08.$$

The regression coefficient matrix **c** between matrix **X** and the respective eigenfrequencies is calculated in a least squares sense according to Eq. (11) and the cost function is minimized, resulting in new parameter values as input for the next iteration step. In the next iteration step, the resonance frequencies for the resulting plate length and width are calculated, the order of the interpolation polynomial is increased by 1 and a second-order interpolation is used to solve the nonlinear least squares problem of minimizing the cost function. By calculating a higher order fit through the already known values of the updating parameters (L and W) and the calculated F.E. frequencies, respectively, the F.E. model is replaced by an interpolation polynomial and needs to be calculated only a few times.

To summarize: the updating algorithm consists of four steps:

(1) construct matrix (12) consisting of combinations of regression order coefficients and updating parameters,
(2) calculate interpolation polynomial between calculated resonance frequencies and their respective updating parameter values (θ),
(3) minimize cost function (10) based on measured and interpolated resonance frequencies, taking into account the standard deviation,
(4) solve nonlinear least squares problem to estimate new parameter value θ, based on the defined cost function, measured (target) eigenfrequencies.

The multiple parameter updating algorithm previously discussed will now be evaluated on the aluminum test plate with known geometric and material properties.

5.2. Updating procedure using two parameters

One can now check if the calculated cost function, minimizing the differences between measured and calculated modal frequencies, is reaching a global minimum for the exact test plate length (L) and width (W), taking into account the variation of the two updating parameters between user-defined boundary value combinations used as starting values. The following results are obtained for updating the test plate length (L) and width (W), taking into account the measurement uncertainty, the first 10 modes and using a very fine mesh (Table 11). Three starting values are chosen for L (between 0.35 and 0.45 m) and for W (between 0.08 and 0.15 m).

Table 11
Updating test plate length (L) and width (W)

	Calculated (m)	Exact (m)	Absolute difference (m)	Relative difference (%)
Length L	0.3922	0.3900	0.0022	0.56
Width W	0.1126	0.1100	0.0026	2.36

One can conclude that the updating errors when updating the plate length and width are not equal. This can be explained by the fact that the length is much higher than the width and the F.E. model accuracy will begin to show. Comparing with the updating results of only one parameter (Tables 4, 5 and 7), the errors, in case of simultaneous updating of both parameters, are of comparable accuracy (Table 12), although the errors are slightly smaller when updating both parameters simultaneously.

5.3. Updating procedure using three parameters

Now one can also try to take into account three updating parameters, length (L), width (W) and Young's modulus (E), and investigate how the updating algorithm is minimizing the nonlinear least squares cost function. As starting values, four user-defined linear interpolated parameter vectors must be specified (Table 13). For instance, one can choose the following upper and lower boundary values for the three updating parameters, based on 10% difference with respect to the measured or theoretical values.

Two different test cases are calculated:

- Updating using default mesh and updating on basis of the first five ('dm m5') and first 10 resonance frequencies ('dm m10').
- Updating using a very fine mesh on basis of the first five ('fm m5') and first 10 resonance frequencies ('fm m10').

The three different calculation cases for the general parameter updating algorithm, taking into account the measurement uncertainty, generate the results presented in Tables 14 and 15.

From Tables 14 and 15, one can draw the following conclusions:

Table 12
Comparison of updating results using different (1par and 2par) updating algorithms

	Updating 1par (m)	Updating 2par (m)	Diff. 1par (%)	Diff. 2par (%)
Length L	0.3931	0.3922	0.79	0.56
Width W	0.1136	0.1126	3.27	2.36

Table 13
Used boundary parameter values as starting values for L, W and E

	Exact	Lower	Upper
L (m)	0.3900	0.351	0.429
W (m)	0.1100	0.099	0.121
E (Pa)	7.00E + 10	6.30E + 10	7.70E + 10

Table 14
Simultaneous plate model updating of three parameters L, W, E using a default mesh

	Exact	dm m5	Relative difference (%)	dm m10	Relative difference (%)
L	0.3900	0.3959	1.51	0.3983	2.13
W	0.1100	0.1163	5.73	0.1139	3.55
E	7.00E + 10	7.23E + 10	3.29	7.30E + 10	4.25
Time (s)		81.31		129.56	

Table 15
Simultaneous plate model updating of three parameters L, W, E using a fine mesh

	Exact	fm m5	Relative difference (%)	fm m10	Relative difference (%)
L	0.3900	0.3963	1.62	0.3941	1.05
W	0.1100	0.1114	1.27	0.1127	2.45
E	7.00E + 10	7.06E + 10	0.79	7.13E + 10	1.81
Time (s)		153.45		307.71	

Table 16
Plate model updating of three parameters L, W, E using 'normal' mesh

Mode	Test (Hz)	dm m10 (Hz)	dm m1–5 (Hz)	dm m6–10 (Hz)
1	404.33	401.10	404.10	392.9
2	820.51	816.60	801.90	796.9
3	1115.16	1106.70	1114.80	1084.1
4	1702.86	1702.60	1675.00	1662.2
5	2185.34	2161.60	2177.20	2117.9
6	2735.45	2720.50	2683.40	2657.1
7	3589.07	3537.40	3560.50	3466.2
8	3948.22	3923.50	3883.10	3834.8
9	5065.04	4876.30	4977.90	5042.6
10	5356.18	5298.30	5274.90	5183.6

- Using a fine mesh leads to better (more accurate) updating results. A coarse mesh generally results in a structure that over-predicts vibration frequency. A fine mesh generally gives an answer closer to the exact solution.
- The mesh effect is more important than the number of modes taken into account. An optimization of the mesh, gives a better, more accurate representation of the structure in reality than taking into account more modes.
- Updating using only the first five modes leads to more or less the same accuracy in updating results for the test plate, even when the standard deviation is taken into account.
- One might expect that when using a coarse mesh, the updating results would be less accurate when taking only a few modes into account for updating the test plate. This is not shown in the previous results (compare 'dm m5' with 'dm m10' in Tables 14 and 15). A possible explanation could be that when using a coarse mesh, the mesh and respective modal frequencies are accurate enough when only the first five modes are used for model parameter optimization. One could wonder if the coarse mesh will be accurate enough when using higher modes (modes 6–10) for optimization.

In Table 16, the optimization procedure is executed for a coarse mesh in three different case:

- updating using a coarse (default) mesh and updating on basis of the first 10 resonance frequencies ('dm m10'),
- updating using a coarse (default) mesh and updating on basis of the resonance frequencies 1–5 ('dm m1–5'),
- updating using a coarse (default) mesh and updating on basis of the resonance frequencies 6–10 ('dm m6–10').

From Table 16 it is clear that the updating results for a coarse mesh are more accurate when using the first five modes than when using modal frequencies 6–10. The coarse mesh will lose accuracy and an increased spatial resolution is required to accurately capture the higher (more complex) mode shapes and respective modal frequencies. The updating results have more or less the same accuracy when using the first 10 modes or only

Table 17
Comparison between estimated test and updated F.E. modes using a fine mesh

Mode	Test (Hz)	FE upd fm m10 (Hz)	FE upd fm m5 (Hz)
1	404.33	404.70	398.40
2	820.51	825.70	824.10
3	1115.16	1116.30	1099.00
4	1702.86	1721.20	1716.20
5	2185.34	2180.10	2146.60
6	2735.45	2748.90	2737.00
7	3589.07	3567.40	3513.90
8	3948.22	3962.70	3939.20
9	5065.04	4936.10	4958.50
10	5356.18	5252.50	5304.90

the first five modes for updating the F.E. model. For the different F.E. calculation cases, the best results are thus obtained with a very fine mesh, updating with the first 10 resonance frequencies as reference frequencies and taking into account the measurement uncertainty (case 'fm m10'). For this F.E. model with updated parameter values L, W and E, the difference in resonance frequencies with respect to the test modes is listed in Table 17.

6. Conclusions

This document describes the development of an automated model updating procedure. The proposed procedure can handle multiple updating parameters and takes into account the measurement uncertainty. Frequency response function measurements were performed on an aluminum test plate with known geometrical and material properties and the resonance frequencies are estimated. The automated model updating technique is described and the updated parameters from the finite element (F.E.) model are compared to the original ones. It is shown that updating the finite model taking into account the measurement uncertainty gives better results. The results are explained and discussed. For the different F.E. calculation cases, the best results are thus obtained with a very fine mesh, updating with the first 10 resonance frequencies as reference frequencies and taking into account the measurement uncertainty. Using a fine mesh leads to more accurate updating results. The mesh effect is more important than the number of modes taken into account. Updating using only the first five modes leads to more or less the same accuracy in updating results for the test plate, even when the standard deviation is taken into account.

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